An ODE-PDE model for Covid-19 pandemic with vaccination and loss of immunity: well-posedness and simulation

Robin Vaudry*[1], Candy Sonveaux[2,3], Didier Georges[4], and Clémentine Prieur[1]

[1]Laboratoire Jean Kuntzmann – Univ. Grenoble Alpes, CNRS, Inria, Grenoble INP, 38000 Grenoble, France – France

[2]Département de Mathématique [Namur] – Belgique

[3]Namur Center for Complex Systems [Namur] – Belgique

[4]Grenoble Images Parole Signal Automatique – Univ. Grenoble Alpes, CNRS, Grenoble INP, GIPSA-Lab, 38000 Grenoble, France – France

Résumé

The health crisis caused by the COVID-19 pandemic showed once again the need to model the propagation of epidemics. Compartmental models remain the dominant tool for meeting this need. The best-known compartmental model to date is the SIR model introduced by Kermack and McKendrick in 1927. Since then, a variety a SIR-like models have been studied.
In this talk, a SEHIRVD model is considered. In this model, at each time the population is divided in seven distinct compartments: the group S of Susceptible individuals, the group E of Exposed individuals that are infected but not infectious, the group H of Hospitalized individuals, the group I of Infectious individuals, the group R of Recovered individuals that cured from the disease, are still immune and have not been vaccinated since they recovered, the group V of Vaccinated individuals that still hold an immunity against infection and the group D of Deceased individuals.

Our model is structured in age of infectivity and in age of immunity. The age of infectivity a is used in compartment I. Formally, I(t,a) is the proportion of individuals at time t that have been infected for a time a. The age of immunity a' intervenes in compartments R and V. It counts the time spent in these compartments, where individuals are immune to infection. The model includes a progressive loss of immunity for both recovered and vaccinated individuals, parametrized by the age of immunity a'.

The system of equations driving the model is a system of coupled ordinary differential equations and partial differential equations with integral terms.

In this work, we state theoretical results on the well-posedness and bounds for the solution of the system.

Then we propose to solve numerically the equations with a discretized scheme, based on

*Intervenant

radial basis function-generated difference (RBF-FD) method to compute the differential operators in age a and a'. The integrals are approximated numerically with a Quasi-Monte Carlo (QMC) quadrature. The time evolution is calculated through a semi-implicit Euler scheme of order one.
Finally simulations allow to compare different scenarios.

Mots-Clés: Epidemiology, sir, age structured, vaccination, rbf, quasi monte carlo